Intermixing over Cell Boundary between Taylor Vortices

KUNIO KATAOKA
and
TAKAHIRO TAKIGAWA
Department of Chemical Engineering
Kobe University
Rokkodai, Kobe 657 Japan

The rate of exchange of fluid elements between Taylor vortices was experimentally determined for low axial flow rates. A model was formulated defining the exchange coefficient appropriate for this flow system. In the laminar vortex flow region, the measured exchange coefficient increases with Re but is almost independent of Ta.

SCOPE

The tangential annular flow between two coaxial cylinders has a transition from the purely laminar to the laminar Taylor vortex flow when the revolution of the inner cylinder exceeds a critical value. If a small axial flow is independently added to this laminar vortex motion, each vortex pair marches through the annulus in single file holding its original shape. The toroidal motion of fluid elements within vortices causes effective radial mixing whereas the cell boundaries prevent fluid elements from being exchanged between vortices, especially when the axial flow rate is small. Each pair of vortices can be considered as a well-mixed batch vessel moving through the annulus with the same residence time. The flow system has the possibility of a locally well-mixed, plug-flow reactor required for slow liquid reactions. The exchange coefficient at the cell boundaries is defined to establish a new model appropriate for this flow system and measured by salt-solution tracer technique.

CONCLUSIONS AND SIGNIFICANCE

There exists a small degree of exchange of fluid elements between pairs of Taylor vortices. The exchange coefficient at the cell boundaries has been defined to establish a new model. The measured exchange coefficient was found to be almost independent of the rotating speed of the inner cylinder and has been correlated by $Sh_B = 3.7 \times 10^2 Re^{1.7}$ in the range $0 < Re < 35$ and $220 < Ta < 660$. The exchange coefficient decreases with decreasing axial flow rate; the flow system approaches a desired plug-flow, whose fluid elements are locally well-mixed but have very narrow residence time distribution.

INTRODUCTION

Any chemical process equipment will deviate to some extent from the ideal cases of perfect mixing and plug flow. The product yields in chemical reactors would be highest if the radial mass transfer were maximized and the axial transfer minimized.

The present paper is an investigation of the inherent mixing properties in an annular flow formed by two coaxial cylinders with rotation of the inner cylinder. When the revolution of the inner cylinder exceeds a critical value, transition occurs from the purely laminar to the laminar vortex flow; there appears a series of counter-rotating cellular vortex pairs oriented axially along the cylinders. From the engineering viewpoint, the mixing problems become very important when the fluid possesses an independent axial motion. If a small constant axial flow is added to this laminar vortex motion, each vortex, whose size is approximately equal to the annular gap, will march through in single file holding its original shape. Kataoka et al. (1975) showed the possibility of an ideal plug-flow reactor in this flow system.

The toroidal motion of fluid elements causes highly effective radial mixing within cellular vortices whereas the cell boundaries prevent fluid elements from being exchanged between vortices. Each pair of vortices can be considered as a well-mixed batch vessel (which will be called mixing unit in this paper) moving through the annulus with a constant velocity. Therefore it can be presumed that all the fluid elements leaving the annulus have the same residence time in the apparatus. If the reaction is exothermic, the energy generated in and removed

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from the reactor should be suitably adjusted so that the reactor operation is stable. In addition, a narrow temperature range of operation is often required to avoid undesirable by-products. The apparatus of this kind can easily be controlled to maintain uniform temperature owing to the radial mixing. The thermal properties of the flow system have also been investigated by Kataoka et al. (1977).

The modes of flow can be generally characterized as a function of two independent dimensionless parameters: Taylor number of the rotating motion and Reynolds number of the axial motion. The flow regime of interest is that of laminar vortex flow with small axial flow.

The purpose of this paper is to measure the rate of exchange of fluid elements over the cell boundaries and establish a model for application to chemical apparatus.

EXPERIMENTAL

The experiment was performed by the use of salt-solution tracer technique. The experimental apparatus, Figure 1, comprises two coaxial vertical cylinders. The inner cylinder, turned from a brass pipe, is 320 mm long with an O.D. of 60.00 ± 0.02 mm. The transparent outer cylinder, made of acrylic resin so as to make possible visual observations, is 360 mm long with an I.D. of 100.00 ± 0.02 mm. The annular space formed by the two cylinders is 20.00 ± 0.04 mm wide and 295 mm long.

The working fluid enters the annular space at a constant axial flow rate through four 8.5 mm-dia. inlet holes of the bottom flange, and leaves through twelve 10 mm-dia. outlet holes drilled in the wall of the outer cylinder near the top flange. The flow rate is measured with an orifice flow-meter located on the inlet pipeline. The test section has four conductivity cells E_1, E_2, E_3, E_4. Each conductivity cell was constructed from a 4 mm-dia. glass rod having 0.5 mm-dia. platinum wire electrode mounted 1.5 mm apart. The four cells are located every 20 mm in the axial direction. The lowest cell, E_1, is at the height of 105 mm above the bottom. Conductivity cells E_2 and E_4 were used primarily as measuring points 1 and 2 for two-point measurement.

The working fluid and the experimental conditions are listed in Table 1. In order to obtain stable laminar vortex flow, the viscosity of the working fluid was increased by adding glycerin. A small amount of NaCl was also added to the working fluid to measure precisely the baseline of NaCl concentration with the conductivity cells.

The injectant is 0.2 k-mol/m^3 concentration of NaCl solution made by adding NaCl to the working fluid. About 0.5 ml of trader solution (which corresponds to about 0.5 vol% of one vortex) was injected as a pulse through the injection point into the inlet pipeline near the entrance.

By passing water at a constant temperature and at a high velocity inside the inner cylinder, the fluid temperature in the annulus was kept uniform at 24.5°C.

The critical Taylor number for Taylor instability was ascertained to be approximately $Ta_{cr} = 57$ with zero axial flow rate for the present geometry $d/R_i = 0.67$. However, if an axial flow is added, the entry length problem may arise where Taylor vortices appear. It has been ascertained by visual observation that the entry length is short enough in the experimental range of Re and Ta. The Taylor number was limited between 220 and 660 in order to obtain stable laminar vortex flow regime in the whole annular space.

MODEL CONSIDERATION

Consider three pairs of vortices in series moving axially with a constant velocity V_z as shown in Figure 2. Each pair of vortices can be regarded as a well-mixed vessel, which will be called a "mixing unit." Since the cell boundary between neighboring mixing units is not perfectly inpermeable, there should exist a small degree of exchange of fluid elements over the cell boundary if the mixing units have different concentrations.

The exchange coefficient k_B of fluid elements over the cell boundary can be defined in a similar manner to the mass transfer coefficient:

$$J = k_B(C_{n+1} - C_n)S \tag{1}$$

Here S is the mass transfer area on the boundary between two mixing units $n+1$ and n, and is approximately equal to the cross-sectional area of ring-shaped annular space $= \pi(R_o^2 - R_i^2)$ since the volume of the boundary layers formed on both cylinder walls is negligibly small compared to the volume of mixing unit.

In the unsteady tracer technique used, the net rate of change in concentration of a questioned mixing unit is given by the mass balance for the nth mixing unit:

$$V\frac{dC_n}{dt} = k_BS(C_{n+1} - C_n) - k_BS(C_n - C_{n-1}) = k_BS(C_{n+1} - 2C_n + C_{n-1}) \tag{2}$$

where V is the volume of a pair of vortices (a mixing unit), being approximately equal to $\pi(R_o^2 - R_i^2)L_u$. The axial length L_u of a pair of vortices can be determined from the average axial velocity V_z and the time period τ between periodic peaks superimposed on the measured response curve. Conductivity cells E_2 and E_4 were selected as the measuring points. As shown in Figure 1, the first measuring point E_2 is located at the height of 125 mm above the bottom, the second point E_4 being located 40 mm downstream from the first one.

The tracer concentration of the nth mixing unit passing a measuring point i is denoted by $C_{n,i}$. The rate of change in concentration of the mixing unit can be approximately calculated from the concentration change during the time $\Delta t = L_m/V_z$ required for the mixing unit to move from measuring point 1 to 2:

$$\frac{dC_n}{dt} \cong \frac{C_{n,2} - C_{n,1}}{\Delta t} \tag{3}$$

Since Taylor vortices occupy fully the annular gap, the average axial velocity calculated from the axial flow rate divided by the cross-sectional area deviates less than 5% from the actual axial velocity of moving mixing units.

The exchange coefficient can be experimentally determined using the following approximate equation:

$$k_B = \frac{L_u\dfrac{dC_n}{dt}}{C_{n+1} - 2C_n + C_{n-1}} \cong \frac{L_u\dfrac{C_{n,2} - C_{n,1}}{\Delta t}}{\overline{C}_{n+1} - 2\overline{C}_n + \overline{C}_{n-1}} \tag{4}$$

where the overline indicates the arithmetic mean between two measuring points:

$$\overline{C}_n = \frac{C_{n,1} + C_{n,2}}{2} \tag{5}$$

For comparison to tubular reactors, the axial-dispersion coefficient is calculated using the following equation (e.g., Levenspiel and Bischoff, 1963):

$$\Delta\sigma^2 = \frac{2}{Pe} = \frac{2D_L}{V_zL_m} \tag{6}$$

where $\Delta\sigma^2$ is the difference between the variances of the experimental response curves obtained at two points.

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 3 shows typical response curves of tracer concentration obtained at two measuring points. The concentration difference ΔC indicates an increase in salt concentration from the baseline.

It can be seen that periodic peaks and valleys are superimposed on the slowly-varying main curve. As in the previous paper by Kataoka et al. (1975), the injection point should have been placed within the annulus to get much steeper peaks of response curve. According to Kataoka et al. (1977), each peak corresponds to a pair of vortices (a mixing unit). Each Taylor vortex occupies fully the annular gap. The boundary layer formed on both cylinder walls is very thin. The cell boundaries can be considered to become thinner at outward secondary-flow part than at inward secondary-flow part. Therefore the vortex arrangement can be shown corresponding to the response curve in Figure 4.

The needle-like peaks appearing in the valley of the response curve suggest the separation of the boundary layer to occur at the inward

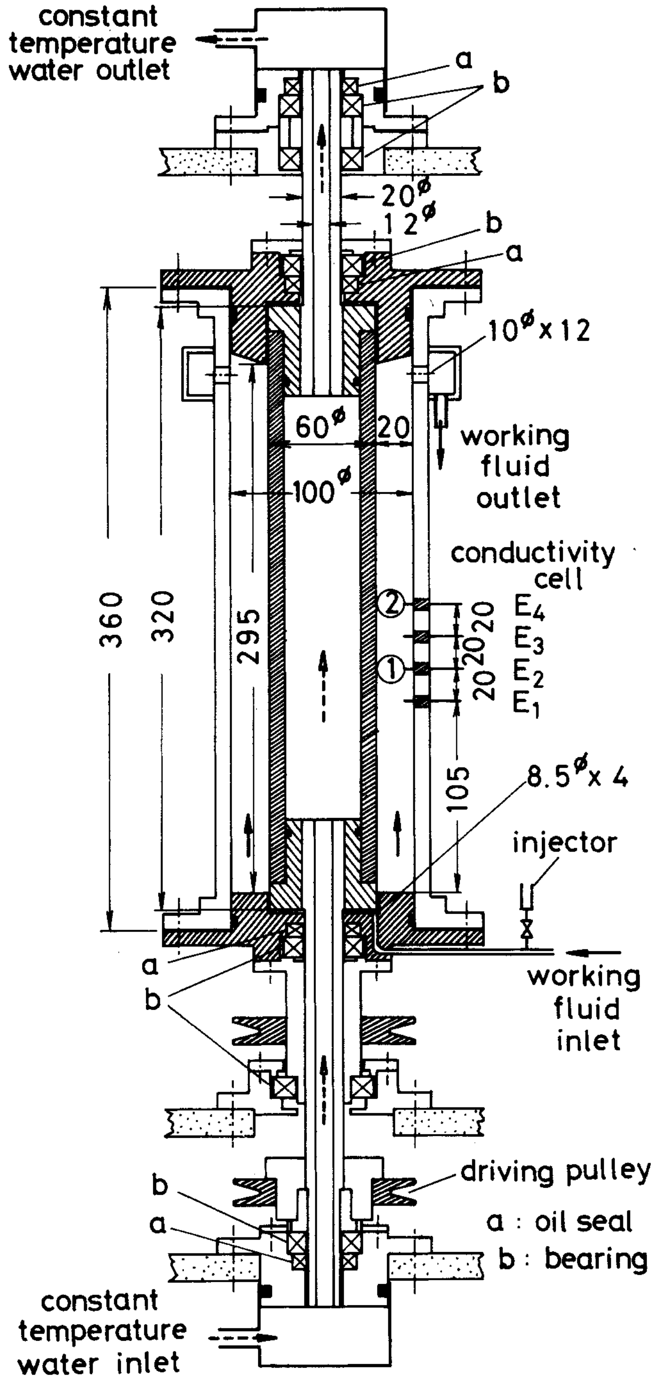

Figure 1. Test section of apparatus. ① : measuring point 1, ② : measuring point 2. Dimensions given are in mm.

TABLE 1. WORKING FLUID AND EXPERIMENTAL CONDITION

Working Fluid:

40 wt.% Glycerin + about 1×10^{-4} k-mol/m^3 NaCl

$\nu = 2.99 \times 10^{-6}$ m^2/s, $Sc = 6900$

Tracer Solution:

40 wt.% Glycerin + about 0.2 k-mol/m^3 NaCl

Experimental Condition:

$V_z = 6.5 \times 10^{-4} \sim 4.27 \times 10^{-3}$ m/s

$Re = 8.73 \sim 57.1$

$Ta = 220 \sim 660$

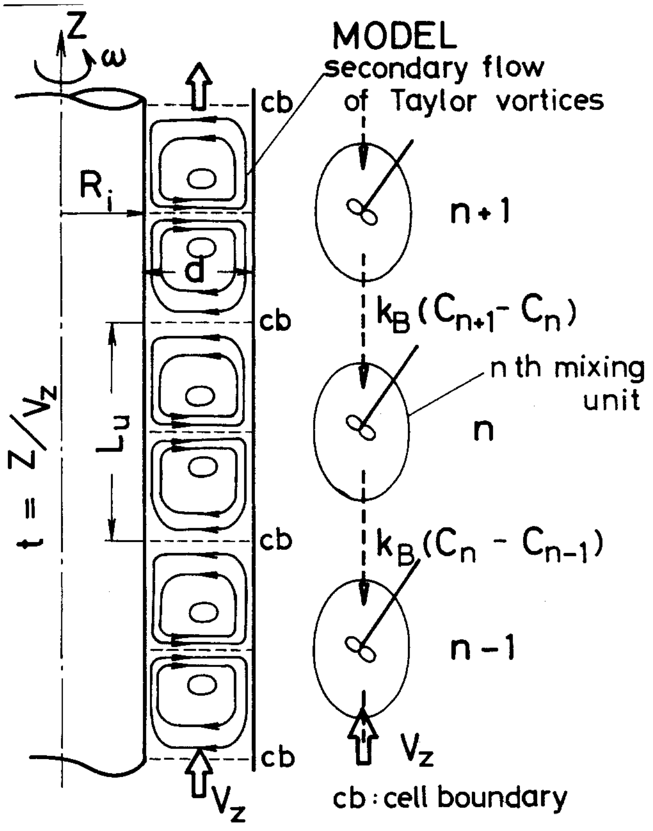

Figure 2. Model for intermixing between Taylor vortex pairs.

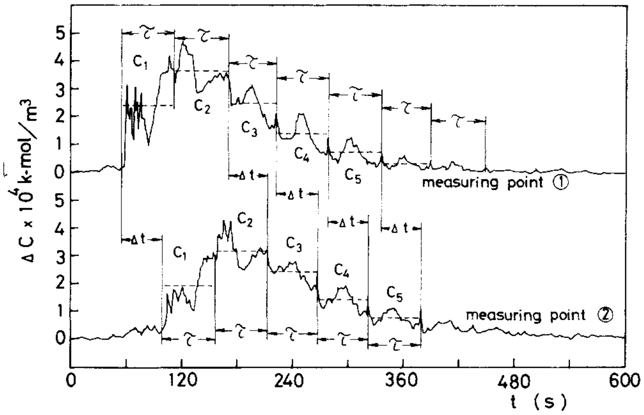

Figure 3. Typical response curves of tracer concentration (Ta = 485.6, Re = 11.9, $L_u = 4.9 \times 10^{-2}$ m, $V_z = 8.9 \times 10^{-4}$ m/s, τ = 55.4 s, Δt = 44.9 s).

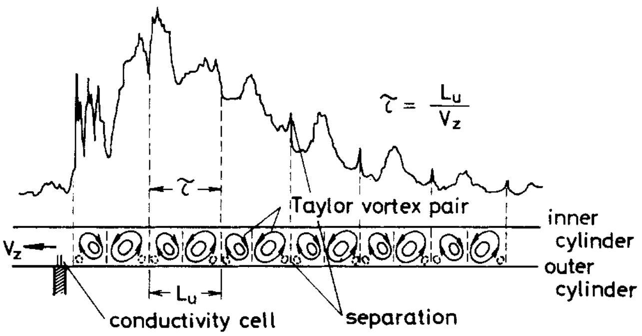

Figure 4. Schematic picture of Taylor vortex arrangement deduced from the experimental response curve.

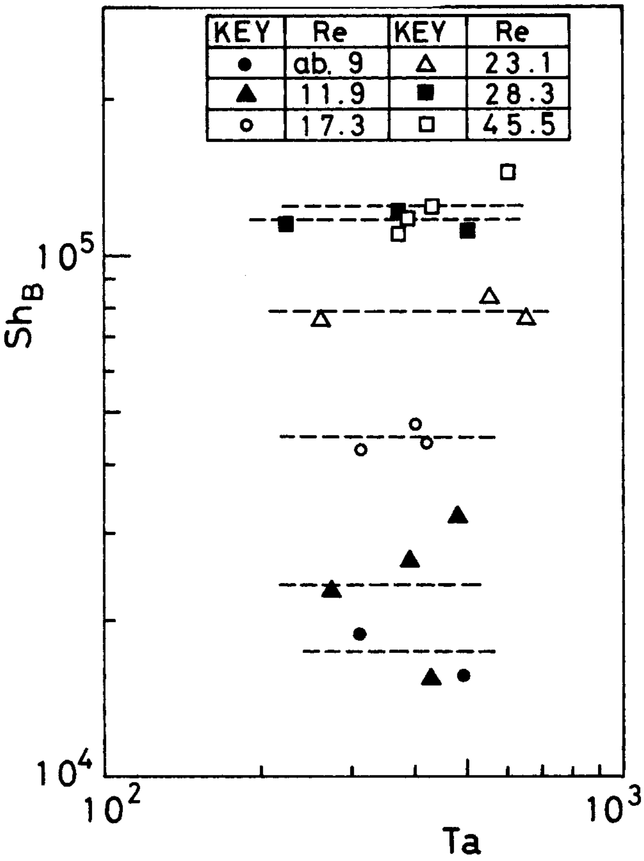

Figure 5. Effect of Taylor number on the dimensionless exchange coefficient (modified Sherwood number) in the laminar vortex flow region.

secondary-flow part. The time period τ corresponding to a pair of vortices is determined from the time interval between the neighboring needle-like peaks. The physical interpretation of Figure 4 is that the mixing effect within pairs of vortices (mixing units) is high due to the toroidal vortex motion, whereas the intermixing effect over the cell boundary between neighboring mixing units is very low, similar to an air curtain.

Within the experimental conditions, the measured exchange coefficients range from 3×10^{-4} to 5×10^{-3} m/s. Although there are no rigid boundaries between neighboring mixing units, a modified Sherwood number based on the exchange coefficient can be defined as:

$$Sh_B = \frac{k_B d}{D} \tag{7}$$

where d is the annular gap, D the molecular diffusivity.

Figure 5 shows the effect of the Taylor number on the Sherwood number. As expected, the exchange coefficient is hardly affected by the Taylor number i.e. the rotating motion in the laminar vortex flow region. It is attributable to the fact that the exchange of fluid elements over the cell boundary occurs solely by molecular diffusion since all the three components of velocity have very small gradients at the cell boundary irrespective of the Taylor number. Figure 6 shows the effect of the Reynolds number on the Sherwood number. It can be seen from the Figure that the exchange coefficient i.e. intermixing effect decreases with decreasing axial flow rate. In order to obtain the desired plug-flow condition, the Reynolds number should be made as small as possible, although the axial flow rate is also related to the residence time or the contact time.

As shown in Figure 7, superimposed peaks on the main response curve disappear when Re exceeds 50, and a smooth continuous response curve is obtained. This indicates that the flow approaches the axially dispersed flow.

For the present apparatus the desired plug-flow regime can be obtained in the range of $0 < Re < 35$ and $220 < Ta < 660$ for $d/R_i = 0.67$. The exchange coefficient can be correlated by the following 'best-fit' equation:

$$Sh_B = 3.7 \times 10^2 \, Re^{1.7} \tag{8}$$

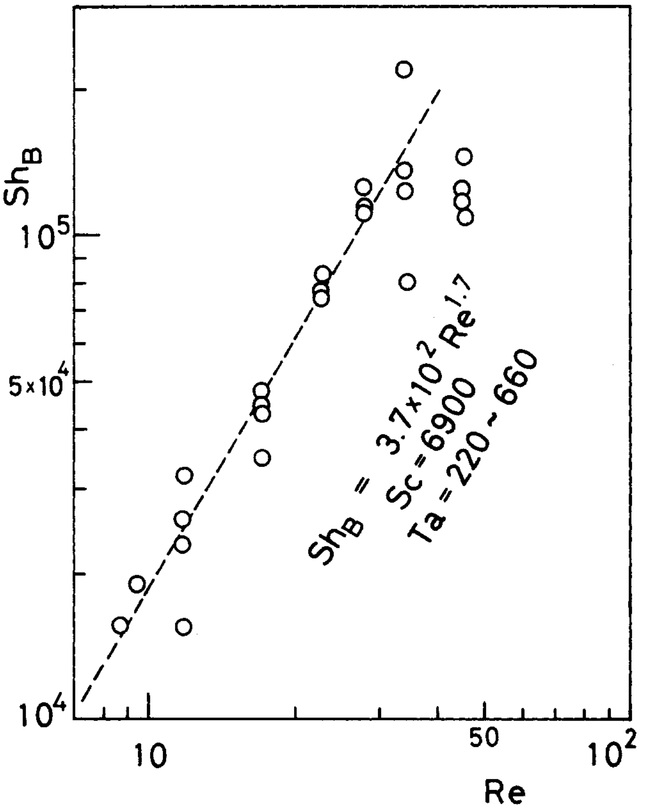

Figure 6. Effect of Reynolds number on the dimensionless exchange coefficient (modified Sherwood number) in the laminar vortex flow region.

which is valid in the range of $0 < Re < 35$ and $220 < Ta < 660$ for $Sc = 6900$.

The Peclet number calculated by Eq. 6 is plotted against the Reynolds number in Figure 8. It is seen that Pe first increases linearly with Re, and decreases rapidly when Re exceeds approximately 30. This indicates that the axial-dispersion coefficient D_L remains constant up to $Re = 30$, but increases rapidly with the axial velocity V_z when Re exceeds 30. Figure 9 shows that the dispersion coefficient remains almost constant independent of the rotating motion although there is some scatter in the data.

According to Levenspiel and Bischoff (1963), the Peclet number of a circular pipe flow is of the order of 10^{-2} for laminar flow of liquid around $Re = 10$, whereas the Peclet number of the present apparatus is of the order of 1.0. Therefore the dispersion coefficient of this flow system can be considered to be 10^{-2} times that of tubular reactors. This is attributable to the extremely small intermixing over the cell boundaries and to the uniform axial velocity distribution over the annular cross section. It should be noted that the exchange coefficient data in Figure 5 show much less scatter than the plots of the Peclet number in Figure 9. This indicates that the present model is more appropriate for the present flow system than the axial-dispersion model.

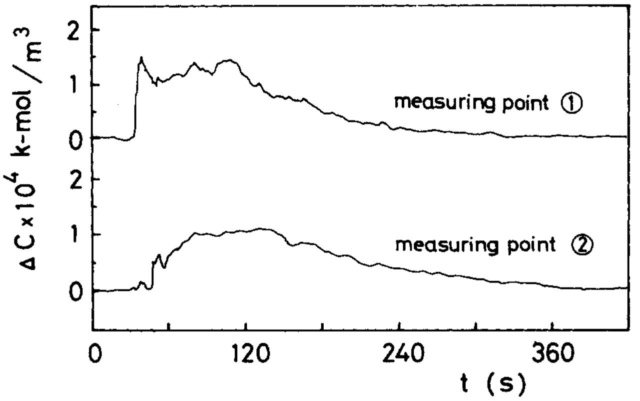

Figure 7. Response curves of tracer concentration for large Reynolds number ($Ta = 407.1$, $Re = 57.1$).

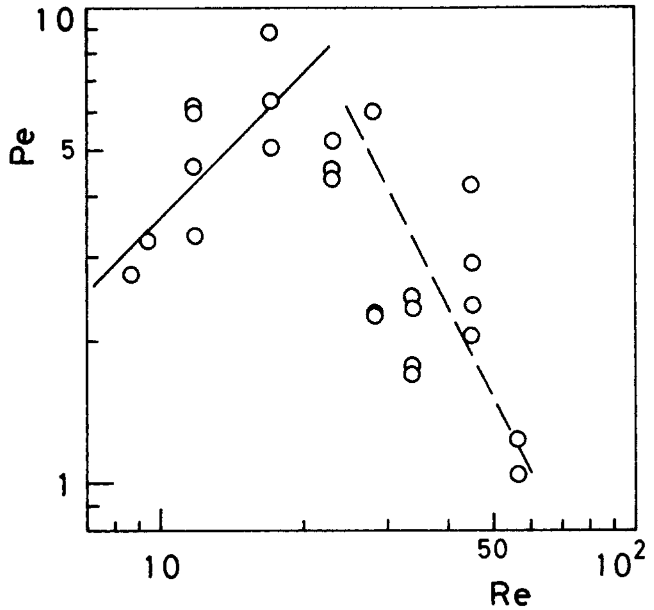

Figure 8. Peclet number vs. Reynolds number.

Once the exchange coefficient is known, the design equation can be applied to chemical reactors. For an mth order chemical reaction with rate constant k_r

$$V\frac{dC_n}{dt} = k_B S(C_{n+1} - 2C_n + C_{n-1}) - k_r V C_n^m \qquad (9)$$

By comparing Eq. 9 with the usual batch reactor equation, it is seen that the additional term, which is associated with the intermixing over the cell boundaries, arises.

The present research did not treat the end effect with the inlet and outlet on the mixing. The next step for application to chemical apparatus is to devise more elaborate structure of the entrance and exit to minimize the undesired mixing. If the tracer had been directly injected into the annulus, a much narrower residence time distribution curve could have been observed. Still this flow system can be beneficial for slow liquid reactions requiring the precise control of contact time if the reactions are restrained in the inlet and outlet pipelines.

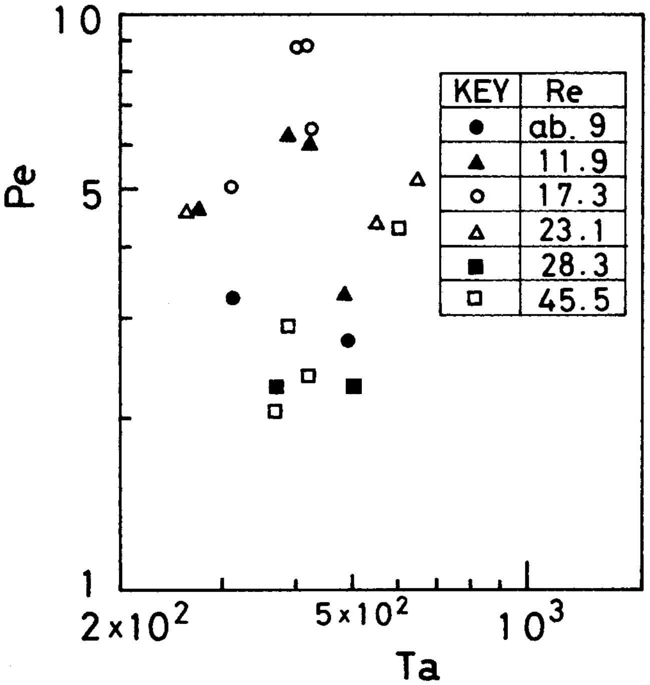

Figure 9. Peclet number vs. Taylor number.

CONCLUSION

If an axial flow is added to laminar Taylor vortex flow, each pair of vortices, which can be regarded as a well-mixed batch vessel, marches through the annular space in single file. However it has been found that there exists a small degree of exchange of fluid elements over the cell boundary between neighboring pairs of vortices. A new model appropriate for the flow system has been formulated defining the exchange coefficient. When the axial flow rate is small enough, the exchange coefficient becomes very small—the flow system approaches an ideal plug flow with no intermixing over the cell boundary. The measured exchange coefficients increase rapidly with *Re* but are almost insensitive to *Ta* in the laminar vortex flow region. The best-fit correlation is given by the equation $Sh_B = 3.7 \times 10^2 Re^{1.7}$ over the range of $0 < Re < 35$ and $220 < Ta < 660$.

NOTATION

C_n = concentration of nth mixing unit, k-mol/m³
ΔC = increase in concentration from baseline, k-mol/m³
D = molecular diffusivity, m²/s
D_L = axial-dispersion coefficient, m²/s
d = annular gap width, m
J = rate of exchange of fluid elements over cell boundary, k-mol/s
k_B = exchange coefficient defined by Eq. 1, m/s
k_r = rate constant of chemical reaction, $(\text{k-mol/m}^3)^{1-m}$/s
L_m = distance between two measuring points, m
L_u = length of mixing unit in the axial direction, m
m = order of reaction
n = number of mixing unit
Pe = Peclet number = $2V_z d/D_L$
Re = Reynolds number = $2V_z d/\nu$
R_i = inner cylinder radius, m
R_o = outer cylinder radius, m
S = mass transfer area at cell boundary, m²
Sh_B = modified Sherwood number (dimensionless exchange coefficient) = $k_B d/D$
Ta = Taylor number = $(R_i\omega d/\nu)(d/R_i)^{1/2}$
t = time, s
Δt = time required for a mixing unit to move from measuring point 1 to 2, s
V = volume of a mixing unit, m³
V_z = average axial velocity, m/s
Z = axial coordinate, m
ν = kinematic viscosity, m²/s
σ = variance, (-)
τ = time required for a mixing unit to pass a point, s
ω = angular velocity of inner cylinder, 1/s

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